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Bulk Viscosity: Past to Present

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A review of the concept of a bulk viscosity coefficient β is presented, involving a discussion of theoretical approaches and a summary of existing experimental data. Two independent viscosity coefficients μ and λ are obtained in the viscous stress tensor as a result of the isotropic Newtonian assumption. Hence, all solutions of the Navier–Stokes equations must make an assumption regarding the functional form of λ. With $\beta = \lambda + \frac{2}{3}\mu$ by definition, the assumption is often arbitrarily made that the two viscosity coefficients are not independent, with $\beta = 0$. This assumption has been shown to be valid only for dilute monatomic gases and is equivalent to stating that dilatational flowfield effects are not significant. Theoretical approaches for quantifying β are classified according to whether the subject fluid is dilute or dense. Experimental methods used to estimate bulk viscosity are described. Existing experimental data are summarized for each fluid classification, and issues related to using these data for dilatational flows in local thermodynamic equilibrium are addressed.

Nomenclature

a = thermodynamic speed of sound, m s^{-1}
c_p = constant pressure specific heat, m^2 s^{-2} K^{-1}
f = frequency, Hz
p = thermodynamic pressure, Pa (atm)
t_c = characteristic flow time, s
α = sound absorption coefficient
β = bulk viscosity coefficient, Pa s
γ = ratio of specific heats
δ_{ij} = Kronecker delta
ε_{ij} = rate-of-deformation tensor, s^{-1}
ε_{kk} = dilatation, s^{-1}
κ = thermal conductivity coefficient, J s^{-1} m^{-1} K^{-1}
λ = second viscosity coefficient, Pa s
μ = shear viscosity coefficient, Pa s
ρ = flow density, kg m^{-3}
σ_{ij} = stress tensor, Pa
υ = relaxation time, s
Ω = β/μ, bulk viscosity ratio

Introduction

THE presence of bulk viscosity terms in the Navier–Stokes equations has been generally accepted for some time. This acceptance is because of the continuing influence of Stokes,[1] who researched the variation of shear stress with strain rate for fluids. In general, the stress tensor for a Newtonian fluid element may be written as

$$\sigma_{ij} = (-p + \lambda\varepsilon_{kk})\delta_{ij} + 2\mu\varepsilon_{ij}$$

We can express compressibility effects associated with λ in a different manner by isolating terms related to the dilatation ε_{kk}. This is accomplished by formulating the average normal stress as

$$\bar{p} = \frac{\sigma_{11} + \sigma_{22} + \sigma_{33}}{3} = -p + \left(\lambda + \frac{2}{3}\mu\right)\varepsilon_{kk}$$

According to Truesdell,[2] Saint-Venant[3] implied that the second term in the preceding equation involving the dilatation could be neglected for the majority of fluid flows. The combination of viscosity coefficients multiplying this term has received a special designation in the literature and is called the bulk viscosity coefficient, defined as

$$\beta = \lambda + \tfrac{2}{3}\mu$$

Stokes made an argument,[1] similar to that of Saint-Venant, that the bulk viscosity could be set to zero. In fact, over the years this assumption has become known as Stokes' hypothesis. Kinetic theory[4] and experiment predict that $\beta = 0$ for a dilute monatomic gas. This result is often adopted, regardless of the nature or internal structure of the fluid, and is equivalent to stating that dilatational effects for a given problem can be neglected. Introductory textbooks in fluid mechanics usually acknowledge that this assumption is not always valid, but nevertheless assume the bulk viscosity to be zero.[5,6]

Any transport property can be related to a nonequilibrium relaxation process. If the departure from thermal equilibrium is very small, this property can be expressed as a function of the local thermodynamic state. To determine when the local thermodynamic equilibrium (LTE) assumption is applicable, one must identify a characteristic flow time t_c and compare its magnitude with the relaxation time υ of each lagging process.[7] If t_c is much larger than υ, then the LTE assumption is satisfied. Hence, it becomes phenomenologically possible to interpret a relaxation process as corresponding to an effective transport coefficient and tabulate this coefficient as a function of thermodynamic properties. This includes general relaxation processes such as dissociation, ionization, evaporation, and chemical reactions. The preceding statements serve only to loosely establish the domain of frequency-independent transport processes, highlighting their problem-dependent nature. The reader is referred to more advanced works[8–10] to establish rigorously the limitations of this approach for a particular problem.

The dissipative process traditionally associated with bulk viscosity for a dilute gas is internal molecular relaxation.[11] In a dense gas, bulk viscosity occurs from the viscous forces that arise when a volume of fluid is compressed or dilated without change of shape,[12] whereas a liquid bulk viscosity involves the structural relaxation associated with rearrangement of molecules during acoustic compression and rarefaction.[13,14] The suggestion has been made that the use of a single parameter to capture the multitude of nonequilibrium effects associated with a complex relaxation process is questionable.[15] However, as long as the LTE condition is satisfied for each dissipative process, the interpretation of bulk viscosity as a transport coefficient is valid.

For larger departures from equilibrium, the interpretation of bulk viscosity as a transport coefficient is no longer appropriate, no more than it would be for the shear viscosity or thermal conductivity. In this event bulk viscous effects must be replaced with general rate

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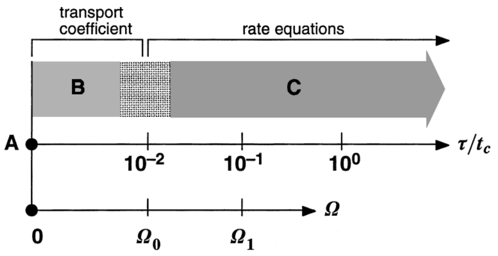


Fig. 1 Relaxation schematic for the bulk viscosity ratio Ω.

equations. Rate equations are not used to model shear viscosity or thermal conductivity effects in the Navier–Stokes equations because the relaxation times associated with these transport processes are extremely small, generally on the order of a few molecular collision intervals. As just implied, if bulk viscous effects are neglected, we must adopt Stokes' hypothesis $\lambda = -\frac{2}{3}\mu$ and use a rate equation to describe each relaxation process, in addition to the mass, momentum, energy, and state equations. This approach can be computationally expensive if there are multiple relaxation processes to consider, highlighting the motivation for bulk viscosity methods, namely enhancing computational performance.

Figure 1 is a schematic illustrating the applicability of bulk viscosity methods relative to the general rate equation approach. Stokes' hypothesis is always valid at point A, which corresponds to a dilute monatomic gas. In region B, the LTE assumption is valid, and an internal relaxation process may be interpreted in terms of a bulk viscosity coefficient. The LTE condition is no longer applicable in region C, where the relaxation time is on the order of the characteristic flow time. Here, one must apply Stokes' hypothesis, and rate equations are used to model translational-internal energy exchange. Note the fuzzy area between regions B and C, implying that the exact time ratio at which the LTE condition is valid is not clear. Associated with each labeled time ratio are values of the bulk viscosity ratio Ω_0 and Ω_1 ($\Omega = \beta/\mu$), whose values are problem dependent. Although not a focus of the present review, we note that as the frequency ratio further increases the frequency-dependent nature of the bulk viscosity coefficient must be considered.

The principal goals of this review are a preliminary collation of existing experimental data for the bulk viscosity coefficient, the bulk viscosity ratio, and the second viscosity coefficient for different fluid classifications and to gain physical insight into the mechanisms responsible for bulk viscous effects in fluid dynamics. Additional emphasis is placed on relating estimates of experimental uncertainty when this information is available. In the next section we outline experimental methods for estimating the bulk viscosity coefficient. We continue with a review of the relevant bulk viscosity literature for each fluid classification, summarize available experimental data, and address issues related to using these data in general dilatational flows. We conclude with a brief summary.

Experimental Methods

Acoustic Absorption of Sound Waves

Acoustic absorption measurement of sound waves is the only demonstrated method capable of estimating bulk viscosity. This method involves considerable potential for error because one measures a total absorption coefficient, then subtracts contributions to the absorption because of the shear viscosity and thermal conductivity. The total absorption coefficient is written as

$$\alpha_{\text{total}} = \alpha_{\text{classical}} + \alpha_{\text{excess}}$$

$$\alpha_{\text{classical}} = \frac{2\pi^2 f^2}{\rho a^3}\left(\frac{4}{3}\mu + \frac{\gamma - 1}{c_p}\kappa\right)$$

or

$$\alpha_{\text{total}} = \frac{2\pi^2 f^2}{\rho a^3}\left(\frac{4}{3}\mu + \frac{\gamma - 1}{c_p}\kappa + \beta\right)$$

Experiments involving bulk viscosity attempt to relate observed excess absorption α_{excess} to β. Errors are introduced in these estimates when different investigators use varying sources for auxiliary data needed to compute the total absorption, such as μ and κ.

Attenuation experiments in ambient air are strongly influenced by frequency and relative humidity. The presence of the H_2O molecule[16] accelerates the vibrational relaxation of N_2 and O_2, and because these molecules are not energetic at room temperature, they have difficulty achieving equilibrium without this added effect. This dilemma complicates efforts to study bulk viscous effects in high-speed air flows.

Alternative Experimental Methods

Alternative experiments to estimate bulk viscosity are identified by following the same approach as the acoustic absorption method. One identifies a flow that exhibits both large Ω and dilatation, and after data reduction correlates an experimentally measured quantity with the bulk viscosity. Experiments that measure shock-wave thickness satisfy this procedure, as the interior of shock waves are known to exhibit large values of dilatation, with large Ω values dictated by the choice of test gas. Emanuel and Argrow[17] deduced a linear relation between shock-wave thickness and β for a dense gas. They numerically modeled SF_6 (sulfur hexafluoride), yielding a shock wave many thousands of mean free paths thick. Given Ω and other data as initial conditions, the shock-wave structure was numerically determined to provide an estimate of the thickness. Thus, Ω estimates are obtained by an inverse method where results from shock-thickness experiments are compared with the output from a numerical procedure. A similar approach, in principle, was taken by Sherman,[18] who experimentally measured shock-wave thickness in dilute air. An additional experimental measure for bulk viscosity has been outlined by Gonzales and Emanuel,[19] who suggested the use of a rotating cylinder with porous walls in Couette flow, with the skin friction measured at the upper wall serving as the primary experimental parameter. To the authors' knowledge, there have been no bulk viscosity estimates obtained from these proposed experimental methods. Of particular interest would be shock-thickness measurements of a large Ω gas, such as carbon dioxide, over a wide temperature range.

Bulk Viscosity of Dilute Gases

In this section, theoretical procedures for estimating the bulk viscosity of a dilute gas are described, followed by experimental data found in the literature. A similar approach is followed in a subsequent section for dense gases and liquids. We shall focus on describing basic principles and concepts rather than detailed calculations, which are both complicated and tentative in nature. Cited references are those that directly mention the bulk viscosity coefficient, the bulk viscosity ratio, or the second viscosity coefficient. Thus, experiments whose sole purpose was observation of excess sound absorption without interpretation are not referenced in the present work. In addition, this review is restricted to theories that deal with the frequency-independent bulk viscosity coefficient in accord with the LTE condition.

Fully aware of the work of Stokes and its implications, Tisza[20] first introduced an additional term into the Navier–Stokes equations to account for the excess absorption observed in experiments involving certain monatomic and polyatomic gases and liquids. Although the primary focus of his investigation was the dissipation arising from the time lag for the establishment of equilibrium of the internal degrees of freedom, similar interpretations may be applied to any system that involves energy distributed over different degrees of freedom. Tisza also discussed the implications of the introduction of bulk viscosity, primarily that it would be important for flows with large dilatations. Tisza recognized the importance of the LTE condition, which is necessary if bulk viscosity is to be used in the Navier–Stokes equations.

Wang Chang and Uhlenbeck[21] developed a semiclassical Boltzmann formulation to obtain general expressions for transport coefficients in a pure polyatomic gas when translational-internal energy transfer is difficult. The concepts of easy and difficult energy exchange were introduced. Easy energy exchange refers to rapid exchange between the translational and internal degrees of freedom, corresponding to small relaxation times. Thus, one temperature is adequate for describing the distribution of energy. Difficult exchange corresponds to longer relaxation times, and separate temperatures are required for the translational and internal modes. Detailed analysis resulted in an expression for the bulk viscosity coefficient written in terms of a collision integral that could not be evaluated.

Karim and Rosenhead[22] performed a literature review similar to the present work concerning the second coefficient of viscosity. Their stated purpose was to identify fluid-dynamic trends implied by the assumption of Stokes' hypothesis. They identified several theories to explain the disagreement between experiment and theory. On the basis of acoustic absorption experiments, they concluded that the application of Stokes' hypothesis beyond dilute monatomic gases was not justified. Gilbarg and Paolucci[23] showed with calculations in normal shock waves that inclusion of a bulk viscosity coefficient in the Navier–Stokes equations produced larger shock widths than investigations that adopt Stokes' hypothesis. No original bulk viscosity data were reported in this work; however, reliable values of the bulk viscosity were not well known for polyatomic gases.

The classical Chapman–Enskog method for solving the Boltzmann equation in a pure gas is a perturbation method in which the equilibrium solution is used as the unperturbed function.[12,24] Curtiss,[25] Curtiss and Muckenfuss,[26] and Muckenfuss and Curtiss[27] developed the multicomponent kinetic theory of nonspherical molecules under conditions near equilibrium applicable to dilute polyatomic gases. A modified Boltzmann equation was solved using an extension of the Chapman–Enskog perturbation scheme in which the effects of the rotational degrees of freedom were considered. Expressions for the transport coefficients, including the bulk viscosity, were developed in terms of multidimensional integrals that depend on the dynamics of the molecular collision process and the assumed molecular model. These expressions for the transport coefficients are applicable to any rigid, nonspherical, convex body in which the center of mass is a center of symmetry, and were subsequently evaluated for sphero-cylindrical molecules.

Sherman[18] measured normal shock profiles and shock widths in a low-density wind tunnel for helium at Mach numbers of 1.72 and 1.82 and for air at Mach numbers from 1.78 to 3.91. The assumption was made that $\Omega = 0$ for helium and $\Omega = \frac{2}{3}$ for air to quantitatively match the experimental shock widths. Talbot and Scala[28] investigated shock structure for a diatomic gas with a single internal degree of freedom. They assumed that the Navier–Stokes equations are applicable inside the shock over a wide range of Mach numbers, comparing results from a rate equation method and a bulk viscosity method. The conclusion was made that the rate equation method was superior to the bulk viscosity method because the latter only produced correct results for very small values of the dilatation. However, no attempt was made to explicitly bracket the range of applicability of the two solution methods. In addition, the fact that the Navier–Stokes equations were extended beyond their accepted domain of applicability could possibly alter the conclusions of this work concerning the utility of the bulk viscosity method.

Monchick et al.[29] extended the single-component gas kinetic theory of Wang Chang and Uhlenbeck[21] to multicomponent mixtures, with the ultimate goal of developing approximate methods for evaluating the various transport coefficients. This formulation assumed rapid equilibration between the translational and internal degrees of freedom (easy energy exchange). The expression relating bulk viscosity to a relaxation time in a multicomponent mixture was shown to be identical to that derived for a pure gas. It was also questioned whether the theory was applicable beyond the LTE condition.

Chapman et al.[30] investigated potential reasons for the failure of the Navier–Stokes equations in hypersonic shock structure at high altitudes for monatomic gases and nitrogen. An attempt was made to model discrepancies between theoretical shock profile data and experiment by adding bulk viscosity terms explicitly to the governing equations. This was the approach even for monatomic gases; however, the validity of Stokes' hypothesis was questioned on the basis of Truesdell's arguments.[2] The numerical procedure involved adjusting parameters in the bulk viscosity terms to match shock-wave density thicknesses obtained from experiment. The numerically predicted shock widths were in good agreement with experiment, but the method was incapable of accurately capturing the temperature profile. Better results were obtained through the addition of a nonlinear Burnett term, an approach pursued in subsequent research.[31] For diatomic nitrogen better agreement was found for both the shock-width estimate and the temperature profile.

Emanuel[32] briefly reviewed bulk viscosity concepts in the context of diatomic and polyatomic dilute gases. In particular, he noted that linear triatomic molecules with characteristic vibrational temperatures near room temperature, such as CO_2 (960 K) and N_2O (847 K), should possess Ω values on the order of 2000. He concluded that bulk viscous contributions might be significant for high-speed entry into planetary atmospheres. Emanuel[33] further studied bulk viscous effects related to the boundary layer of a high-speed flow of CO_2 over a flat plate. His numerical procedure assumed values for Ω on the order of 1000, with the analysis showing that bulk viscous effects can significantly influence heat transfer under hypersonic re-entry conditions. Gonzales and Emanuel[19] examined the effect of bulk viscosity in a Couette flow of CO_2 with porous walls. An Ω value of 2000, characteristic of CO_2, was assumed, and the transverse density variation was compared with a similar profile obtained assuming Stokes' hypothesis, indicating significant differences. This study also described an experimental procedure capable of estimating bulk viscosity as described in an earlier section.

Orou and Johnson[34] investigated a flow situation where the characteristic relaxation process was generated by turbulence. An expression describing the rate of entropy production in a reacting flow was developed to relate the second viscosity coefficient to a relaxation time. Ern and Giovangigli[35] developed a kinetic theory method for the calculation of the rotational bulk viscosity. Approximate solutions were obtained by solving linear systems of equations based on the Chapman–Enskog expansion method using iterative procedures. Test calculations were performed using an air mixture and a 16-component, combustion-oriented methane mixture to demonstrate the utility of these approximate expressions. An extension of the theory to include the vibrational bulk viscosity was also described.

Meador et al.[15] challenged the interpretation of bulk viscosity as a relaxation parameter. They developed a method applicable to small-signal sound propagation through a dilute diatomic gas. Their method generalized that of Wang Chang and Uhlenbeck[21] and Monchick et al.[29] beyond the restriction of $\upsilon \ll t_c$, and thus by definition is not a bulk viscosity method. The resulting expression for the speed of sound was contrasted with a similar relation developed from a method based on reaction kinetics. The discrepancy between these two relations, reflecting the apparent inability of the bulk viscosity method to predict the speed of sound at high frequencies, was the primary evidence presented to support the claim that β is not a relaxation parameter.

Emanuel[36] developed a perturbation theory for the absorption and dispersion coefficients and entropy production that holds for large values of Ω and satisfies the LTE condition. Comparisons between an exact formulation and the low-frequency limit show good agreement for a large Ω gas (CO_2). Emanuel emphasized the importance of the LTE condition and mentioned that it was unclear when this condition is satisfied in the sound absorption literature.

Despite the number of theoretical investigations that predict bulk viscosity in dilute gases, in reality, there are very few reports of actual experimental data. Tisza[20] reported $\Omega = 2000$ for CO_2 and N_2O based on the results of Kneser,[37] and for air that shear and bulk viscosity are of comparable magnitude. Prangsma et al.[38] reported bulk viscosity data obtained from sound absorption measurements for N_2, CO, CH_4, and CD_4 (tetradeuteromethane) over a temperature range of 77–300 K. A test run performed using neon at $T = 77$ K gave $\beta = 0$, verifying Stokes' hypothesis for monatomic gases. The data obtained in this investigation indicated that both β and Ω increase with temperature for each gas over the specified temperature range. Other data presented involved reviews of the sound absorption literature for N_2 near 300 K, with limited data obtained for the other gases. Overall, trends indicate that β was roughly on the order of μ for N_2 and CO, whereas more complex molecules have Ω values that are greater than unity. The reported experimental error in the determination of β was 10%.

Ash et al.[39] estimated the magnitude of the second viscosity coefficient from sound absorption measurements in air over a temperature range of 284–323 K and relative humidity range of 6–91%. As mentioned in the Experimental Methods section, the presence of the angular H_2O molecule in an air mixture is known to accelerate the vibrational relaxation of oxygen and nitrogen. Reduction of the sound absorption measurements indicated that the ratio λ/μ varies from 2000 to 20,000 over the given temperature and humidity range. The conclusion was made that the analysis supported the idea of bulk viscous effects resulting from molecular relaxation processes.

Bulk Viscosity of Dense Fluids

In dense gases and liquids many theories concerning the bulk viscosity involve solutions of modified forms of the Boltzmann equation. Many of these theories are tentative and subject to argument, and so our emphasis will be on theories that have been contrasted with experiment. Classical kinetic theory based on the Boltzmann equation assumes that molecules possess no internal structure and that the gas is dilute. Snider and Curtiss[40] developed the kinetic theory for a moderately dense gas. The modified theory extends the density range over which the Chapman–Enskog procedure is valid and provides expressions for the transport coefficients corrected for higher density. Hanley and Cohen[41] compared the predictions of this modified Enskog theory (MET) for bulk viscosity with experiment. Results indicate that MET captures the temperature and density dependence of the bulk viscosity quite well for dense gases and liquids, excluding anomalous behavior near the critical region.

Rice and Allnatt[42] and Allnatt and Rice[43] developed a kinetic theory for dense fluids which postulated that the intermolecular potential between two molecules has a very short-range strongly repulsive region and a long-range soft portion. They further postulated the existence of two characteristic types of momentum and energy transfer associated with these two regions. For the strongly repulsive region there are large momentum and energy transfers, whereas for the soft portion there are small transfers. Although applicable to dense gases, the Rice–Allnatt kinetic theory was primarily developed for application to liquids. Gray and Rice[44] developed a bulk viscosity relation for a simple dense fluid based on this kinetic theory. For liquid argon at a density of 1.12 g/cm^3 and over a temperature range of 128–185 K, they report $\Omega = 1.3$ (±20%).

Dense fluids near the gas/liquid critical point have received brief attention. Cramer and Crickenberger[45] investigated the dissipative structure of finite-amplitude, single-phase shock waves in fluids with large specific heats. They noted the general scarcity of experimental data or analytical relations for the bulk viscosity and emphasized the general lack of transport data for the heavy fluorocarbons used in the study. Their approach was to assume constant values for Ω and observe the qualitative effect on their results.

Available Experimental Data

The only source of dense gas data for the bulk viscosity is the work of Madigosky for argon.[46] Acoustic absorption measurements were performed over a frequency range of 5–85 MHz, and as a function of density from 0.5 to 1.0 g/cm^3 at a temperature of 235 K. Results indicated that the bulk viscosity is essentially a function of the square of the density, with Ω ranging from 0.25 to 0.7 (±10–20%). Overall, good agreement with the Rice–Allnatt theory was indicated.

In an independent attempt to verify the Rice–Allnatt kinetic theory for liquids, Naugle and Squire[47] performed a series of ultrasonic absorption measurements (±5%) in liquid argon from 30 to 70 MHz at a temperature of 85 K and a pressure of 1.1 atm. Their results predict a nominal Ω range of 0.4–1.2, giving acceptable agreement with the Rice–Allnatt theory. Naugle[48] carried out additional measurements for argon over the frequency range of 30–70 MHz for a pressure of 8 atm and temperatures between 84 and 112 K. He estimated $\Omega = 0.7$ (±50%) at 84, 87, and 90 K, and $\Omega = 1.4$ (±50%) at 112 K, concluding that Ω is approximately independent of temperature at constant pressure. The conclusion was also made that the results agreed quantitatively with the numerical calculation of Gray and Rice.[44] Naugle et al.[49] extended the bulk viscosity estimates for liquid argon to high pressures over a temperature range of 85–145 K. This investigation produced Ω estimates of 2.3–0.8 (±25%) over the indicated density range, and the conclusion was made that the bulk viscosity is essentially a function of the square of the density. Singer[50] measured ultrasonic absorption in liquid methane (CH_4) over a temperature range of 94–146 K. Error estimates for β were on the order of ±25%. The bulk viscosity β increased slightly as density increased, but Ω decreased. The general conclusion reached for this series of experiments on liquid argon and methane is that the transport process responsible for excess absorption is structural relaxation, with the bulk viscosity being a function of the density squared, validating the Rice–Allnatt theory for liquid transport.

Experimental data for liquid metals are available. Jarzynski[51] estimates $\Omega = 4.5$ (±13–20%) in bismuth at frequencies of 68 and 92 MHz over a temperature range of 573–723 K. Similar estimates for mercury were obtained over a lower temperature interval of 298–473 K, with $\Omega = 1.3$ (±19–42%). Hunter et al.[52] carried out experiments for mercury at frequencies of 90, 150, and 270 MHz over a temperature range of 298–403 K, reporting $\Omega = 0.45$. No mention of uncertainties in β or Ω was made, but a 2% error was reported for the absorption measurements. This Ω estimate conflicts with that of Jarzynski,[51] which the authors attribute to different sources for the physical constants used to calculate the absorption coefficient. Letcher and Beyer[53] performed sound absorption experiments in sodium and potassium between 12 and 75 MHz from their melting points to 423 K. The authors conclude that no excess absorption was observed for sodium, but that a small absorption (±50%) existed for potassium with $\Omega = 1.7$–3.6. The conclusion drawn from these experiments for liquid metals is that measured excess absorption cannot be attributed to molecular relaxation effects, but is due to structural relaxation.

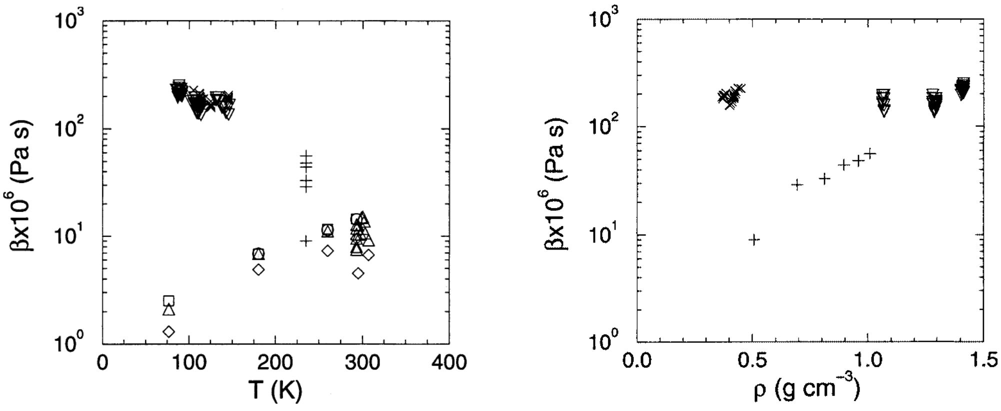
Discussion

Bulk viscosity experimental data discussed in the previous sections for dilute gases and dense fluids are shown in Fig. 2 plotted vs thermodynamic variables. The purpose of Fig. 2 is to show a temperature or density dependence in the bulk viscosity coefficient or the bulk viscosity ratio, similar to the well-established temperature dependence of the shear viscosity coefficient. In panels (a) and (b) β and Ω are shown vs temperature. The bulk viscosity β increases with temperature for dilute gases, but decreases with temperature for dense gases and liquids. The ratio Ω increases with increasing temperature for dilute and dense fluids. However, for dilute gases this temperature dependence is very weak. These results suggest that the nature of the bulk viscosity mechanism has a decided effect on the variation of bulk viscosity with temperature (i.e., the data cannot be represented by a single curve fit). Panels (c) and (d) show β and Ω vs density. No density-dependent bulk viscosity data for dilute gases were found in the literature. In panel (c) β increases with increasing density for dense fluids. In panel (d) the bulk viscosity ratio for dense gases exhibits a weak density dependence and is roughly on the order of the shear viscosity. No relevant trend is observed for the liquid bulk viscosity data in panel (d). Note that the liquid metal data from Refs. 51–53 are not shown in Fig. 2 to prevent clutter.

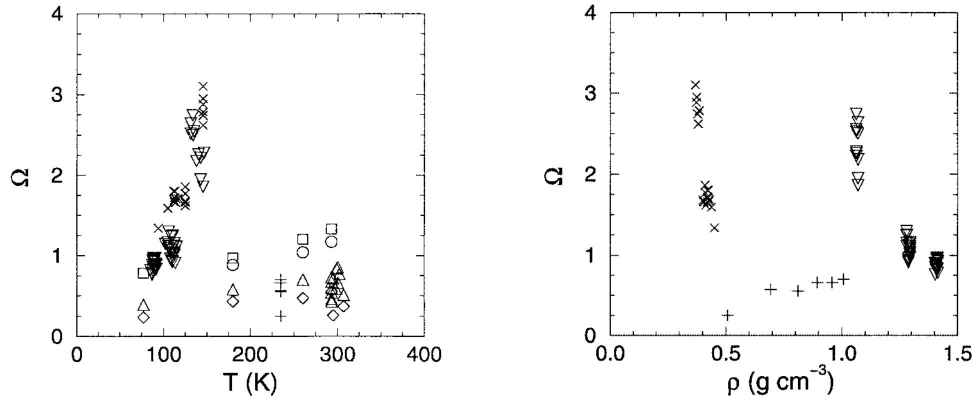
We emphasize that the observed trends are to be treated as tentative. It has not been established that the sound absorption data satisfy the LTE condition. Additional experimental data are needed to gain confidence in these observations, especially for dilute and dense gases.

Bulk Viscosity Issues for General Fluid Dynamic Flows

As illustrated in preceding sections, small quantities of bulk viscosity experimental data exist for various fluid classifications, with the dominating contribution coming from liquids. These data are determined experimentally by relating an excess acoustic absorption to a transport coefficient. Logically, the next goal is to establish



a) *β* **vs temperature** **c)** *β* **vs density**



b) Ω vs temperature **d) Ω vs density**

Fig. 2 Experimental data for the bulk viscosity coefficient *β* **and the bulk viscosity ratio Ω: ○, dilute CD_4[38]; □, dilute CH_4[38]; ◇, dilute CO[38]; △, dilute N_2[38]; +, dense Ar[46]; ▽, liquid Ar[47–49]; and ×, liquid CH_4.[50]**

the validity of these results for dilatational flows that satisfy the LTE condition. Marcy has attempted a first step toward this goal[54] by outlining a procedure for estimating the second viscosity coefficient from sound absorption or dispersion measurements. He noted that bulk viscosity results obtained from acoustics have not been extended to general fluid flows because of the different assumptions made by researchers in acoustics and fluid dynamics. In fluid dynamics the difference between the fluid pressure and the thermodynamic pressure is assumed to be proportional to a constant dilatation, which is consistent with the LTE condition. However, this assumption is not a requirement in acoustics, where complex, frequency-dependent solutions exist for plane waves. This statement suggests that eventually engineering solutions to the Navier–Stokes equations must be able to implement frequency-dependent transport properties.

Conclusions

A contemporary review of the bulk viscosity literature has been presented, covering the variation in density characteristic of dilute gases, dense gases, and liquids. The accepted technique for measuring bulk viscosity is excess sound absorption, which in general produces estimates that are subject to large experimental uncertainties. Alternative experiments have been identified that are capable of providing additional bulk viscosity data for dilute and dense polyatomic gases with improved accuracy. A critical aspect of any bulk viscosity database will be the systematic application of shear viscosity and thermal conductivity measurements in reducing numerical and sound absorption data. To use bulk viscosity experimental data in conjunction with the Navier–Stokes equations, the condition of local thermodynamic equilibrium must be satisfied.

An examination of the literature reveals a distinct lack of bulk viscosity experimental data for dilute and dense fluids. The lack of data for gases characteristic of terrestrial planetary atmospheres, such as oxygen, nitrogen, and carbon dioxide is critical and needs to be remedied. Detailed theories exist for all fluid classifications that predict reasonable agreement with available experimental data, with the exception of dense gases possessing large specific heats.

Bulk viscosity methods seek to improve on the computational performance of existing numerical schemes where dilatational effects are important, but departures from thermodynamic equilibrium are small. Fundamental research is required to precisely define the nature, validity, and range of applicability of this approach to solving problems in fluid dynamics. Successful research using these methods is predicted to lead to engineering estimates of a transport coefficient similar in nature to the ordinary shear viscosity, but restricted to certain classes of physical processes.

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